Reviews of modern interest rate models and their implements

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Abstract

We survey the prevailing interest rate models in the article. The models discussed have their unique advantages as well as their limitations. We include Vasicec models, Hull's modified model, n-factor model, and famous market models. We also discuss the implements of these models.

Key words: Vasicec, Hull's model, n-factor model, market models.

Interest rate derivatives are instruments whose payoffs depend on the dynamics of interest rates. In the last two decades, the volume of trading in interest rate derivatives has been increasing exponentially. Moreover, there are many exotic instruments developed to meet specific needs of different market participants including investors seeking to make gains from their views on the level of future interest rates and hedgers attempting to control their risks in future interest rates. The market participants are keen on studying and developing robust financial models for the pricing and hedging of a broad range of interest rate derivatives.

To tackle the problems of the evaluation and hedging of the interest rate derivatives, [Vas77] proposed a instantaneous spot rate dynamics models. To mend the drawbacks of the Vasicek models, [CIR85] and [HW90b] offered their modified models. A lot of term structure models including [BK91],[LS92], [HW93], etc were proposed to meet specific demands. We'll discuss these models in the next section.

Exponential affine term structure models is one of the oldest and the most widely studied class of dynamic interest rate models. The main advantage of these models is the fact that the yields can be expressed as affine functions of the short rate (or instantaneous spot rate). The exponential affine term structure models are often classified into three categories:

- Gaussian affine models. The single factor linear model proposed in [Vas77] is a Gaussian affine model and was the first model for which closed-form formulae for bond prices were obtained. All the state variables in these types of models have constant volatilities. A multi-factor Gaussian affine model is discussed in [BN99]. Extensions of the Gaussian affine models to match the current term structure are discussed in [HW90b], [HW93] and [HW94]. The Gaussian models have a high degree of tractability and a variety of products can be priced in closed-form with these types of models. Recently, closed-form formulae for swaption pricing under a multi-factor Gaussian affine model have been reported in [SP06b].
- CIR affine models. Models of this type were first proposed in [CIR85] and were extended to multi-factor case in [BT91]. All the state variables in these models have CIR-type square root volatilities. Unlike the Gaussian models, the interest rate is guaranteed to remain non-negative provided it starts from a non-negative value.
- A three-factor affine family. This family represents the models that mix Gaussian and CIR type state variables; see [BDS96], [Rhe99] and [LS92]for examples.

A general framework for multi-factor affine term structure models was proposed in [DK96].

Important interest rate models

[Vas77] proposed a path-breaking instantaneous spot rate dynamics model, which gave arbitrage-free prices for bonds and bond options. The Vasicek model describes the dynamics of the short rate in a linear equation and the short rates can be solved explicitly. The bond price can be expressed in a simple closed form depending on the parameters and the initial short rate $r0$. Once all the bond prices of different maturities are known, the entire term structure of interest rates can be known. However there are a number of disadvantages in the Vasicek model:

- The model might assume negative interest rates with positive probability.
- The Vasicek model cannot reproduce satisfactorily a given term structure.

To improve the Vasicek model, [CIR85] proposed a model which guaranteed positive interest rates. [HW90a] extended the Vasicek model to conform to the initial yield curve by introducing time-varying parameters. [BK91] proposed a model that is a humped-volatility short rate model providing good fitting quality to the market data as well as a positive short rate.

Two-factor models have been proposed to deal with the problem of perfect correlation of different rates in a one factor model. Famous two-factor models include [Ric78], [LS92] and [CS92]. [HW93] proposed a two-factor model, which is able to match the initial yield structure and remove the undesirable perfect correlation of a one factor model.

[BDFS96] proposed a three-factor model (BDFS). The model is highly flexible, giving rise to hump- and spoon-shaped yield curves, however the lack of closed form formulae for the BDFS model is its disadvantage. This model can however still be used to derive joint moments of the short rate at different points in time. [DR04] proposed a two-factor dynamic model with time-varying market prices of risk. [BN98] proposed an n-factor Vasicek model and solves it explicitly for bond prices. [SP06a] has suggested a closed-form formula for pricing Bermudan swaptions using n-factor Vasicek model.

There are advantages to model the interest-rate by the instantaneous short rate. In particular, we may have great freedom in selecting the related dynamics. For example, we are free to select the drift and instantaneous volatility coefficients in the related diffusion dynamics for one-factor short rate models. However, there are some disadvantages for short rate models. For example, it is difficult to obtain an exact calibration to the initial curve of term structure and a clear expression of the covariance structure of forward rates. [HL86] proposed an important model describing the evolution of the term structure in a binomial-tree model rather than in short rate models. Their idea then spurred [HJM92] to develop a general framework for the modeling of continuous time interest rate dynamics (HJM). In general, under the HJM model, we assume that, for each T, the forward rate $f(t; T)$ evolves according to an SDE where the drift term is completely determined by the choice of the diffusion coefficient. The importance of HJM theory is that virtually any (exogenous term structure) interest rate model can be derived within such a framework. However, the key disadvantage of the HJM framework is that it cannot be represented as recombining trees. In practice, this means that it must be implemented by Monte Carlo simulation.

Another class of models called market models have become popular in the recent years. These models include LIBOR Forward Model (LFM) and LIBOR Swap Model (LSM). The main reasons for the popularity lie in their consistency with the market practice of pricing caps, floors and swaptions by the means of Black's formula. The market models propose a coherent frame-work for the joint modeling of a whole set of forward rates instead of short rate. These models have been developed successfully in a series of papers by [MSS97], [BGM97] and [Jam97]. Before these market models were proposed, there were no interest rates dynamics consistent with either Black's formula for swaptions or with Black's formula for caps. However, it can be shown that, given an LSM, the LFM rates will not be lognormal. Moreover, both the LSM and LFM cannot be represented as recombining trees. In practice, it implies that we have to use Monte Carlo simulation to implement the models.

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